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Mass Optimisation of the Vibroinsulating Frame of a Short Vibrating Conveyer.

Introduction.

Vibratory conveyers are applied in industry for a continuous transport of loose materials at short distances, usually up to 20 m. The main disadvantage of such transport is the transmission of significant dynamic forces to the foundation. Due to this feature conveyers are often supported on expensive, heavy frames softly vibroinsulated from the foundation. In the case of long conveyers, of several meters length, the most difficult construction problem constitute natural vibrations of a trough – occurring as bilateral bending in the symmetry plane of the conveyer – disturbing the transport procedure. The detailed analysis of long vibratory conveyers was given in papers [1, 2, 3, 4].

The analysis of short conveyers, of the length of some meters only, placed on the frame vibroinsulated from the foundation - will be presented hereby. It is generally assumed, at the designing stage of vibroinsulating frames, that the frames should be much heavier than the trough of the conveyer. The fact that this assumption is not based on scientific grounds will be proved in the paper. Since the production cost as well as the cost of soft suspension of the frame significantly increases with the increase of its mass – optimisation of this parameter seems reasonable and economically justified.

To verify whether the vibroinsulating frames are performing properly the vibratory conveyer of typical parameters was analysed. In the first part of the hereby paper the vibratory conveyer without the vibroinsulating frame, while in the second part the conveyer supported on the vibroinsulating frame is analysed.

1. **Forces transmitted to the foundation by the vibratory conveyer – without the insulating frame.**

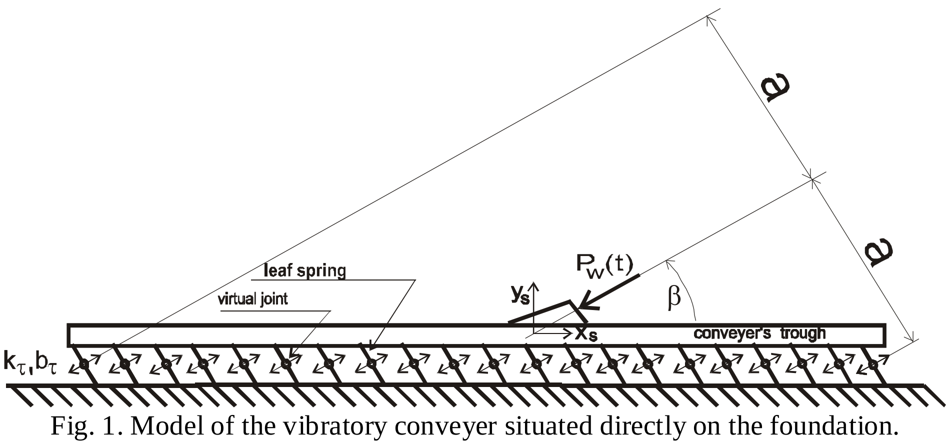

Fig. 1. Model of the vibratory conveyer situated directly on the foundation.

Forces transmitted to the foundation by short vibratory conveyers situated directly – via the system of leaf springs – on the foundation are related to the deformation of elastic elements (Fig. 1). The total value of the foundation reaction, in the direction of vibrations $F(t)$ for the trough of mass m supported by uniformly distributed system of springs - of the elasticity constant k and damping constant b – which forms β angle with the horizontal line and is excited for vibrations by the vibrator of a directed input force $P_w(t)$ (causing the vibrations of the trough of the amplitude A) equals in the steady state:

$$F_\tau(t) = A\sqrt{k_\tau{}^2 + b_\tau{}^2\omega^2} \cdot \sin(\omega t + \gamma) \qquad (1)$$

Thus, the maximum value of the foundation reaction– in the steady state – equals:

$$F_{\tau\,\max} = A\sqrt{k_\tau{}^2 + b_\tau{}^2\omega^2} \qquad (2)$$

In practice, in the steady state, when the system operates outside the resonance zones, the damping of the system has insignificant influence on the force transmitted to the foundation. Thus:

$$F_{\tau\,\max} \cong A \cdot k_\tau \qquad (3)$$

For the parameters of the analysed, typical vibratory conveyer, where A = 0.00241[m], K = 1531750[N/m], the maximum force transmitted to the foundation is:

$$F_{\tau\,\max} \cong 3700[N] \qquad (4)$$

2. Forces transmitted to the foundation by the vibratory conveyer – supported on the insulating frame

At the designing of the vibroinsulating frame the following aspects should be taken into account:

- Direction of the exciting force should pass through the centre of gravity of the trough and the frame as well as through the centre of gravity of the suspension's elastic forces.
- The frame should be relatively massive in order not to be significantly excited in the working direction since that would disturb the transport of material.
- Amplitude of the dynamic force transmitted to the foundation should be much smaller than the one of the force transmitted by the conveyer situated directly on the foundation.
- Static bending due to the material weight should not disturb the "technological procedure".
- The system – in the steady state – should not operate in the vicinity of resonance zones.

2.1 Analysis of the system: trough – vibroinsulating frame

To estimate the effectiveness of the vibroinsulating frame applied in short conveyers the model of the system presented in Fig. 2 was analysed.

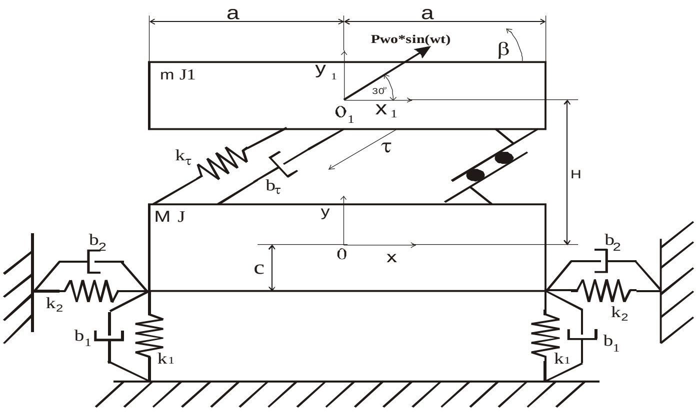

Fig. 2. Schematic presentation of a vibratory conveyer placed on a stiff frame suspended elastically

The presented above scheme corresponds to the mathematical model given by the equation:

$$M * \frac{\partial^2}{\partial t^2} X = Q \tag{5}$$

Matrix M and vectors X and Q are given by the following formulae:

$$M = \begin{bmatrix} M+m & 0 & -m\cos(30^0) & -mH \\ 0 & M+m & -m\sin(30^0) & 0 \\ -m\cos(30^0) & -m\sin(30^0) & m & m*H*\cos(30^0) \\ -mH & 0 & m*H*\cos(30^0) & J+mH^2+J_1 \end{bmatrix} \tag{6}$$

$$X = \begin{bmatrix} x & y & \tau & \beta \end{bmatrix}^T \tag{7}$$

$$Q = \begin{bmatrix} -2\beta k_x c - 2xk_x - 4\dot{x}b_x - 2\dot{\beta}b_x c - F_w\cos(30^0) \\ -2yk_y - 2\dot{y}b_y - F_w\sin(30^0) \\ -\tau k_\tau - \dot{\tau}b_\tau + F_w \\ -2xck_x - 2\beta a^2 k_y - 2\beta c^2 k_x - 2\dot{x}cb_x - 2\dot{\beta}(b_y a^2 + b_x c^2) \end{bmatrix} \tag{8}$$

where:

m, J_1 – mass and the mass moment of inertia calculated versus point 0_1 of the trough,

M, J - mass and the mass moment of inertia calculated versus point 0_1 of the vibroinsulating frame,

k_τ, b_τ - total elasticity and damping of the leaf springs in the working direction τ,

$2k_1$, $2k_2$ - elasticity coefficients of the suspending system of the vibroinsulating frame in direction y and x,

$2b_1$, $2b_2$- damping coefficients of the suspending system of the vibroinsulating frame in direction y and x, The remaining parameters of the system are marked directly in Fig. 2.

The simulation of steady state was performed for the parameters of the typical vibratory conveyer, where: mass of the trough = 3700 [kg], working frequency = 16.7 [Hz], amplitude of the trough = 2.4 [mm] and the amplitude of input force P_{wo} = 90 [kN].

2.2. Selection of the mass of the vibroinsulating frame

When the vibroinsulating frame is suspended in the way shown in Fig. 2, the maximum vibration amplitude of the mass centre of the frame – in the working direction – can be determined by the vibration amplitude in x and y direction:

$$A_{\max} = \sqrt{x^2 + y^2} \tag{9}$$

The vibration amplitude in the direction of operation (τ) of the input force P_{wo} equals:

$$A = y\sin(\alpha) + x\cos(\alpha) \tag{10}$$

where α is the angle between the direction of the input force operation and the horizontal line. The angle $\alpha = 30^o$ in the conveyer being analysed.

For typical parameters of the conveyer $A \cong A_{max}$ while, in the specific case, when $k_1 = k_2$, A equals A_{max}.

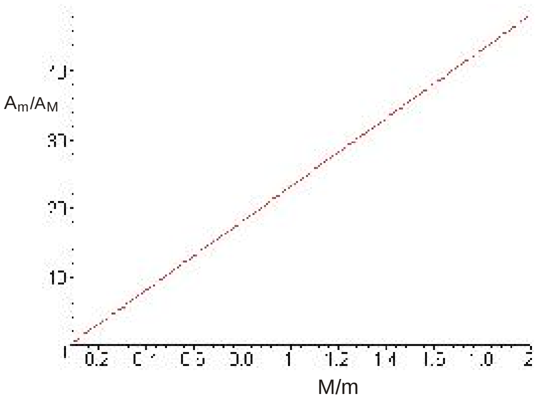

Fig. 3. Ratio of the vibration amplitude of the trough, in direction τ, to the vibration amplitude of the vibroinsulating mass as a function of the ratio of masses: the frame to the trough

The ratio of the vibration amplitude of the trough, in direction τ, to the vibration amplitude of the vibroinsulating mass as a function of the ratio of masses: the frame to the trough (in the steady state) – is presented in Fig.3. It should be noted that the elasticity coefficients k_x and k_y are depending on the mass of the vibroinsulating frame M, – according to the formula:

$$k_x = k_y = k_\tau \frac{M + m}{m} \tag{11}$$

The satisfactory ratio – being over ten – of the vibration amplitude of the trough to the one of the vibroinsulating frame is obtained for the mass ratio equal 0.45, which means for the frame mass being over 1665 [kg] – when the mass of the trough equals 3700 [kg].

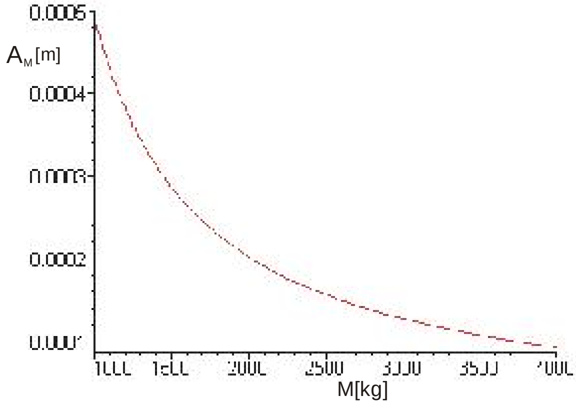

Fig. 4. Amplitude of the vibration of the vibroinsulating frame as a function of its mass

The amplitude of the vibration of the vibroinsulating frame is presented in Fig. 4 as a function of the mass of the frame. It can be seen that at small masses of the frame the amplitude of its vibration is large, due to the vicinity of the resonance zone. When that mass increases its vibration amplitude reduces significantly in the working direction, which in turn reduces the force F_r transmitted to the foundation via the suspension system of the vibroinsulating frame (Fig. 5).

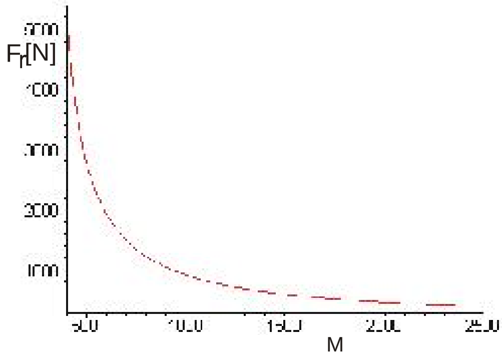

Fig. 5. Amplitude of the force transmitted to the foundation versus the mass of the frame

As can be seen from the graph presented in Fig. 5 the amplitude of the force transmitted to the foundation at steady state for the mass of the frame being 1665 [kg] equals 540 [N], which constitutes 14.4% of the amplitude of the force transmitted to the foundation by the conveyer not placed on the vibroinsulating frame. When the mass of the frame exceeds 2600 [Kg], that being 70% of the mass of the trough, more than 10 times reduction of the force transmitted to the foundation is achieved.

2.3. Amplitude of the vibration of the trough of the conveyer

The amplitude of the vibration of the trough – in the working direction – should not differ significantly from the amplitude required by the proper performance of the technological process. This condition has to be taken into account when designing the vibratory conveyer, supported on the vibroinsulating frame. Fig. 6 presents the amplitude of vibration of the trough versus the ratio of the masses: frame to trough. This amplitude is practically constant, regardless of the mass of the vibroinsulating frame, however, under the condition, that the system operates outside the resonance zones.

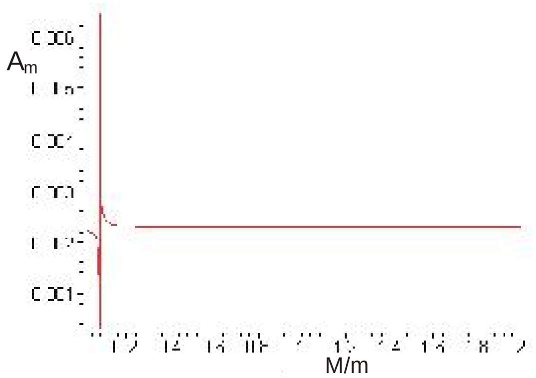

Fig. 6. Amplitude of the vibration of the trough versus the ratio of the mass of the frame to the mass of the trough.

2.4. Static deflection of the vibroinsulated conveyer

When the vibratory conveyer is supported on the vibroinsulating frame the static deflection of the conveyer increases. It requires consideration, when large amounts of material are transported, since there might be problems with the height difference at the ends of the conveyer.

The static deflection caused by the mass of the material, $m_{n,}$ in the conveyer, in which leaf springs are directly fixed to the foundation – equals respectively:

- in x direction: $$A_{xstat} = \frac{m_n g \cdot \sin\alpha \cdot \cos\alpha}{k_\tau} \qquad (12)$$

- in y direction: $$A_{ystat} = \frac{m_n g \cdot \sin^2\alpha}{k_\tau} \qquad (13)$$

This is caused by the fact that the force related to the mass of the material can be projected in two directions - as shown in Fig. 7.

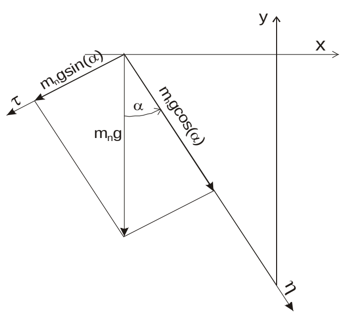

Fig. 7. Distribution of forces related to the mass of the material

The force acting in η direction, which is the axial direction of leaf springs, does not cause any static deflection, while the force acting in the direction of movement τ causes the static deflection, $A_{\tau stat} = m_n g \cdot \sin\alpha / k_\tau$, which - after projecting on x and y directions – gives values A_{xstat} *and* A_{ystat}.

In the case of the conveyer supported on the vibroinsulating frame the static deflection in x direction remains unchanged, while in y direction equals:

$$A_{ystatw} = \frac{m_n g \cdot \sin^2\alpha}{k_\tau} + \frac{m_n g \cdot m}{k_\tau \cdot (m+M)} \tag{14}$$

The graph in Fig. 8 presents the ratio of the static bending in y direction – caused by the mass of the material – of the conveyer placed on vibroinsulating frame to the static bending of the conveyer without the frame versus the mass of the frame. Stiffness coefficient of the frame support is given by equation 11.

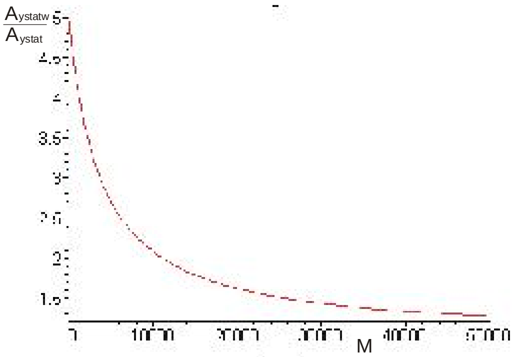

Fig. 8. Ratio of the static bending caused by the mass of the material to the static bending of the conveyer without a frame versus the mass of the frame.

As can be clearly seen from the graph the static bending of the conveyer placed on the vibroinsulating frame might cause problems at large loads of the transported material. For the conveyer being analysed – of the mass of the frame equal 1665 [kg] – the static bending in y direction is 3.7 times higher than the static bending of the conveyer without the vibroinsulation.

2.5. Vibration frequency

In order to determine analytically the frequency of natural vibrations of the system presented in Fig. 2 certain simplifications were necessary. Assuming that damping of the system is small, k_x=k_y and the direction of the force passes through the centre of gravity and the centre of the suspension system, the system of the conveyer can be presented as the one given in Fig. 9.

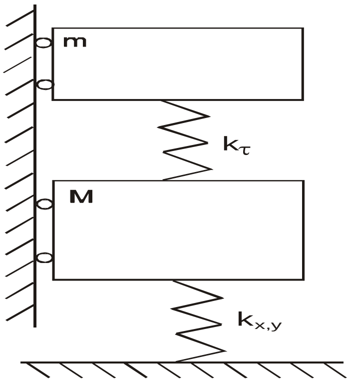

Fig. 9. Schematic presentation of the simplified system: through- vibroinsulating frame

The system has two frequencies of natural - not damped – vibrations:

$$\omega_{1,2} = \sqrt{\frac{1}{2}\left(\frac{k_\tau + k_y}{M} + \frac{k_\tau}{m}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_\tau + k_y}{M} + \frac{k_\tau}{m}\right)^2 - \frac{k_\tau \cdot k_y}{M \cdot m}}} \qquad (15)$$

Taking into account equation 11:

$$\omega_{1,2} = \sqrt{\frac{1}{2}\left(\frac{k_\tau + k_\tau \frac{M+m}{m}}{M} + \frac{k_\tau}{m}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_\tau + k_\tau \frac{M+m}{m}}{M} + \frac{k_\tau}{m}\right)^2 - \frac{k_\tau^{\,2}(M+m)}{M \cdot m^2}}} \qquad (16)$$

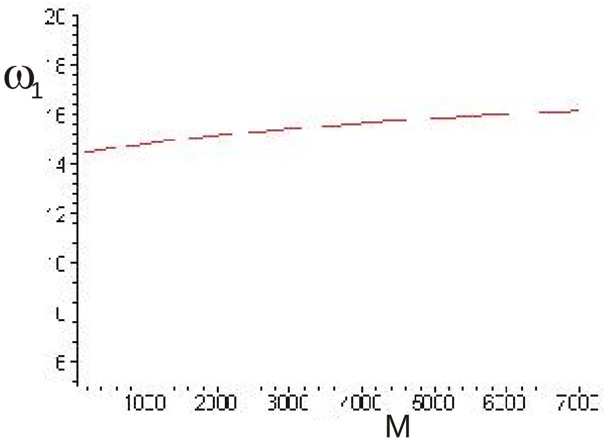

Fig.10. The first natural frequency of the system as a function of the mass of the vibroinsulating frame

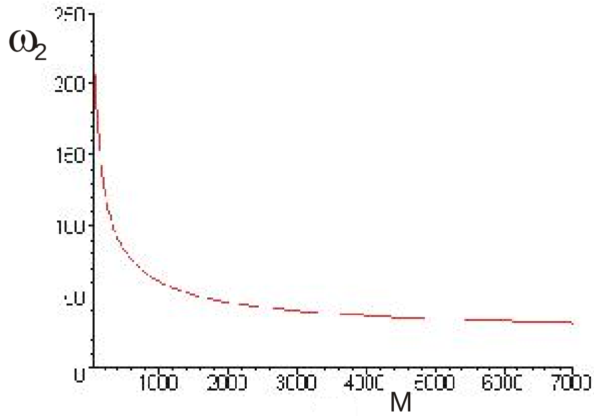

Fig. 11. The second natural frequency of the system as a function of the mass of the vibroinsulating frame

Fig. 10 and 11 present the dependence of the frequency of vibrations of the system on the mass of the vibroinsulating frame. It is seen – from the graphs – that the first natural frequency is not hazardous for the machine operation since it is much lower than the working frequency:

$$\omega_r = 16{,}7 \cdot 2 \cdot \pi = 105 \left[\frac{rad}{s} \right] \qquad (17)$$

The second natural frequency is high for small masses of frames. In the case of the parameters of the analysed conveyer, at the mass of the frame equal 1665 kg, the second natural frequency of free vibrations ω_2 is 49.5 [rad/s], which means that it is sufficiently low to not disturb the machine operation.

When the mass of the frame is going to infinity, ω_1 and ω_2 are going to 20.3 [rad/s], which is the value of the frequency of natural not damped vibrations of the system without the vibroinsulating frame.

3. Conclusions

Taking into account the calculations performed for the typical vibratory conveyer of the mass of the trough being 3700 [kg] the practical possibilities of vibroinsulation by an application of a massive vibroinsulating frame, can be assessed as follows:

1. The satisfactory ratio of the vibration amplitude of the trough to the amplitude of vibration of the vibroinsulating frame is achieved when the ratio of masses: frame to trough – equals 0.45, which means the mass of the frame being above 1665 [kg].
2. When the vibroinsulating frame is applied (even of a very small mass) the amplitude of the trough vibrations in the working direction does not differ significantly from the amplitude required by the technological process.
3. Serious problem of vibroinsulating conveyers constitutes their stiffness bending caused by the mass of the transported material. However, it should be mentioned here that even large increase of the mass of the frame would not decrease significantly the static deflection.
4. Natural frequencies of the system, when the vibroinsulating frame of a very small mass was applied, are appearing dangerously close to the forced frequency. At the increasing mass of the frame the higher frequency of natural vibrations of the

system decreases thus, moving away from the resonance zone. For the proposed frame of a mass of 1665 [kg] the second natural frequency of free vibrations constitutes less than 50% of the forced frequency.

Thus, the hereby-presented research proves that there is no real reason for applying massive frames for vibroinsulation of the vibrating conveyers.

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